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A study of holographic renormalization group flows in $d = 6$ and $d = 3$

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Abstract: We present an explicit study of the holographic renormalization group (RG) in six dimensions using minimal gauged supergravity. By perturbing the theory with the addition of a relevant operator of dimension four one flows to a non-supersymmetric conformal fixed point. There are also solutions describing non-conformal vacua of the same theory obtained by giving an expectation value to the operator. One such vacuum is supersymmetric and is obtained by using the true superpotential of the theory. We discuss the physical acceptability of these vacua by applying the criteria recently given by Gubser for the four-dimensional case and find that those criteria give a clear physical picture in the six-dimensional case as well. We use this example to comment on the role of the Hamilton-Jacobi equations in implementing the RG. We conclude with some remarks on AdS_4 and the status of three-dimensional superconformal theories from squashed solutions of M-theory.

Keywords: Field Theories in Higher Dimensions, M-Theory, Supergravity Models.

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Contents

1. Introduction and summary

The study of the AdS/CFT correspondence has been the major occupation of the hep-th-community since the initial breakthroughs of [1]. Amongst the many aspects of the correspondence, one of the most intriguing is the possibility of formulating the field theoretical RG flow in terms of the classical dynamics of the gravitational theory in the bulk [2, 3].

In studying the RG flow induced by certain operators from an ultraviolet (UV) fixed point, one needs to have a dictionary associating each operator to the appropriate field in the bulk. This requires resolving some possible ambiguity [4] arising for a specific range of bulk masses near the stability bound [5], which amounts to making a *choice* between two theories, both in principle described by the same bulk fields. Usually, only one such theory is supersymmetric and the allowed values for the conformal weight Δ can be read off from the representations of supersymmetry. After the association is made, one still needs to determine whether one is deforming the UV lagrangian by adding the operator itself or simply going to a different vacuum where the operator acquires a non-zero vacuum expectation value (vev). (For a clear exposition of this point see [6].) In view of this physical interpretation, not all the solutions to the gravity theory are acceptable, for instance one should rule out flows in which a positive definite operator acquires a negative vev. We will shortly discuss these conditions.

In a related development, it was noticed in [7] that a formulation of gravity in first order hamiltonian formalism provides further insights into the RG.[1] The double way of writing the equations — one as a second order lagrangian system supplemented by the zero energy constraint and the other as a first order system written in terms

[1]That the RG equations can be given a hamiltonian structure was noticed some years ago [8].

of a "superpotential" — is the origin of some confusion in the implementation of the holographic RG. The name superpotential in this context is somewhat of a misnomer because, whereas the Hamilton-Jacobi equations have a continuum set of solutions parameterized by the constants of motion, only one such solution can be regarded as the superpotential arising in a supersymmetric theory of gravity. We shall reserve the name *superpotential* for the truly supersymmetric one and call all the solutions to the Hamilton-Jacobi equation *generating functions*.

In many circumstances, the few particular generating functions that can be found explicitly are precisely those that can be thought of as true superpotentials. If we are interested in flows between two fixed points of which only one (typically the UV one) appears as an extremum of the known superpotential, we cannot use the first order equations for this purpose and must revert to the lagrangian system. In this case, it is impossible to obtain an analytical solution and it is impossible to resolve the vev/deformation ambiguity by asymptotically expanding it near the UV fixed point. How can one decide then which of these two cases is realized? Also, does the fact that the known generating function cannot be used to connect two fixed points mean that it is useless in connection with the RG flow? Or, if it can be used, how can the same boundary operator induce different flows?

These questions have been addressed in above cited literature and a coherent picture has emerged. So far, most of the attention has been focused on the case of various gauged supergravities in $d = 5$, which are, of course, the ones most relevant to four-dimensional field theories. We will address and solve these problems very explicitly for a particularly simple example — $\mathcal{N} = 1$ $d = 7$ gauged supergravity [9]. This example has all the features we want to study and its simple field content allows for a clear-cut solution.[2] By presenting a thorough and explicit solution of the problem, we hope to contribute in clarifying some points that might have remained obscure from the previous discussions.

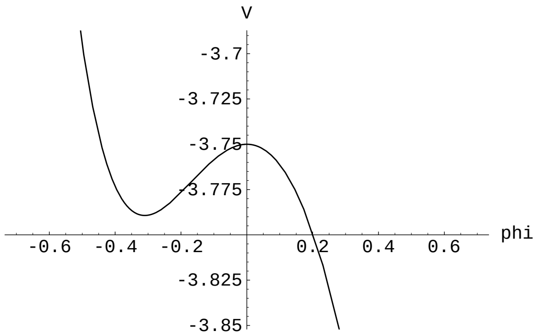

Figure 1: The potential $V(\phi)$ plotted against ϕ. The point $\phi = 0$ represents the UV theory and the point $\phi = -\log 2/\sqrt{5} \approx -0.309985$ the IR theory.

The status of $\mathcal{N} = 1$ $d = 7$ gauged supergravity can be summarized as follows: first of all, there is only one scalar field ϕ in the bulk and its potential has two extrema (see figure 1), a maximum at $\phi = 0$ corresponding to a supersymmetric UV theory and a minimum at[3] $\phi = -\log 2/\sqrt{5}$, corresponding to a non supersymmetric

[2] The case of $\mathcal{N} = 1$ $d = 7$ gauged supergravity has been recently discussed in [10], where some comments in the direction of the results of this paper have been made.

[3] In appropriate units to be specified later.

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but nevertheless stable IR theory. The "tachyonic" excitation near the UV point has a mass m given, in units of the AdS radius r, by $m^2r^2 = -8$. The boundary operator corresponding to ϕ is $\mathcal{O}_\phi = \Phi^2$, where Φ is a scalar in the tensor multiplet of the $d = 6$ CFT or, better, its still unknown non-abelian generalization. The conformal dimension of $\mathcal{O}_\phi$ is $\Delta = 4$. The other possibility ($\Delta = 2$) is ruled out by looking at the [11, table 1] for the multiplets of extended ($\mathcal{N} = 2$) supersymmetry and [12, figure 2]. In fact, $\Delta = 2$ corresponds to the singleton field Φ itself.

Deforming the UV theory ($\phi = 0$) by the addition of $\int \phi\, \mathcal{O}_\phi$ to the fixed point lagrangian induces an RG flow that ends at the non-supersymmetric IR conformal fixed point. In this case the generating function cannot be obtained explicitly but it can be computed numerically and shown to have the correct behavior at both ends — it corresponds to a particular one among the 1-parameter family of solutions of the Hamilton-Jacobi equation. This is the *only* solution in which the field ϕ is allowed to acquire negative values. All other such solutions correspond to *negative* vev's for $\mathcal{O}_\phi$ and should be ruled out, in tune with the fact that, when evaluated on solutions that are running away to $-\infty$, the potential is not bounded from above and the metric singularity at the runaway point is that of a naked time-like nature [13].

It turns out that the only solution to the Hamilton-Jacobi equation that has an extremum and is analytic at $\phi = 0$ is the superpotential. The superpotential can be used to study new supersymmetric vacua of the theory, for which $\langle \mathcal{O}_\phi \rangle > 0$, by studying runaway solutions in which $\phi \to +\infty$. There is also a continuum set of solutions, still with $\phi \to +\infty$, describing what we believe are consistent non-supersymmetric vacua.

Towards the end of the paper we will turn to the more complicated case of compactification of $d = 11$ supergravity [14] on "squashed" manifolds ($\tilde{S}^7$ and $\tilde{N}(1,1)$) and comment on some particular features of the models, complementing the discussion in [15].

In [15], the interesting question was raised of whether there exist trajectories connecting the squashed solutions with the corresponding unsquashed manifolds. The situation is similar to the well-studied case of gauged $d = 5$ supergravity, but there is a crucial difference: in $d = 5$ supergravity, the analog particle rolls from a saddle point to a minimum of the (inverted) potential, whereas here it should roll from a maximum to a saddle point, clearly a more unstable situation. If the RG equations where truly first order, one could argue from general theorems that there must still be a critical line connecting the points. However, the equations expressed in terms of the potential are second order and there is no guarantee that such a solution will survive. In fact, we have reasons to believe that such a flow does not exist, although more work is required to fully establish or refute this belief.

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As far as the squashed solutions are concerned, one is able to find an explicit solution[4] to the Hamilton-Jacobi equation. It turns out that this corresponds to giving a vev to the squashing operator, thus breaking conformal invariance. Given the simple form of the generating function and the collected experience with similar models, it is tempting to conjecture that such a solution is in fact supersymmetric, although here we are working beyond the gauged supergravity truncation and considering fields from the higher levels of the Kaluza-Klein spectrum.

2. $\mathcal{N}=1$ $d=7$ gauged supergravity

The field content, lagrangian and supersymmetry transformations for $\mathcal{N}=1$ $d=7$ gauged supergravity can be found in [9]. For our purposes, we set all fields to zero except for the metric and the scalar ϕ. The action is

$$S=\int d^7x\,\sqrt{g}\left(\frac{1}{2}R-\frac{1}{2}(\partial\phi)^2-V(\phi)\right). \tag{2.1}$$

The scalar potential is chosen to be[5]

$$V(\phi)=\frac{1}{4}e^{-8\phi/\sqrt{5}}-2\,e^{-3\phi/\sqrt{5}}-2\,e^{+2\phi/\sqrt{5}}. \tag{2.2}$$

A plot of $V(\phi)$ is shown in figure 1. There is a supersymmetric UV fixed point at $\phi=0$ and a stable non-supersymmetric IR one at $\phi=-\log 2/\sqrt{5}$. The lagrangian equations of motion following from (2.1), with the standard domain-wall ansatz

$$ds^2=dy^2+e^{2A(y)}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu\,,\qquad\text{and}\qquad \phi=\phi(y) \tag{2.3}$$

can be derived by the following action for a mechanical system:

$$S=\int dy\;e^{6A}\left(15\,\dot{A}^2-\frac{1}{2}\dot{\phi}^2-V(\phi)\right). \tag{2.4}$$

When supplemented by the zero energy constraint, the equations read[6]

$$\ddot{\phi}+6\,\dot{A}\dot{\phi}=V'(\phi)\,, \tag{2.5}$$

$$5\,\ddot{A}+15\,\dot{A}^2+\frac{1}{2}\dot{\phi}^2=-V(\phi)\,, \tag{2.6}$$

$$15\,\dot{A}^2-\frac{1}{2}\dot{\phi}^2=-V(\phi)\,. \tag{2.7}$$

Equation (2.6) can be easily shown to follow from (2.5) and (2.7).

[4]The explicit generating function has no fixed point at the unsquashed vacuum (or else a solution connecting the two would exist).

[5]In its full generality, the potential depends on two arbitrary constants h and g and it displays two minima as long as $h/g>0$ (c.f. [9, 15]). One combination of h and g is eliminated by shifting ϕ and the remaining one is an irrelevant overall multiplicative constant in front of the potential.

[6]The primes denote the derivative with respect to ϕ and the dots the derivative with respect to y.

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Equivalently, one can consider the equation for Hamilton's characteristic function $F(A,\phi,c)$ generating the canonical transformations to the cyclic coordinates[7]

$$\frac{1}{60}\left(\frac{\partial F}{\partial A}\right)^2-\frac{1}{2}\left(\frac{\partial F}{\partial \phi}\right)^2+e^{12A}\,V=0\,. \tag{2.8}$$

By substituting the ansatz $F(A,\phi,c)=e^{6A}\,W(\phi,c)$ into (2.8) the equation becomes the same as the defining equation for the superpotential. Altogether, expressing the canonical transformation in terms of W we end up with the first order system of Hamilton-Jacobi equations

$$\dot\phi = W'\,, \tag{2.9}$$
$$\dot A = -\frac{1}{5}\,W\,, \tag{2.10}$$
$$V = \frac{1}{2}\,W'^2-\frac{3}{5}\,W^2\,. \tag{2.11}$$

Equation (2.11) is obeyed by the superpotential of the theory but it also admits a continuum of solutions, parameterized by c, that have nothing to do with supersymmetry. If one wants to recover all the solutions to the lagrangian equations this way, one needs to consider all possible solutions to (2.11).

Particularly confusing is the fact that there are different solutions to (2.11) that have an extremum at $\phi=0$. One solution, $W_{\rm susy}$, can be easily found by inspection and identified with the superpotential: [8]

$$W_{\rm susy}=-2\,e^{\phi/\sqrt5}-\frac{1}{2}\,e^{-4\phi/\sqrt5}\,. \tag{2.12}$$

The flow between the two fixed points is generated by another solution, $W_{\rm ir}$, not supersymmetric and not analytic at $\phi=0$ that can only be found numerically. The two functions are plotted for comparison in figure 2. The function $W_{\rm ir}$ is rather tricky to find directly from (2.11) but it can be constructed a posteriori once the solution to the lagrangian system (2.5), (2.6) and (2.7) has been found numerically. Such a solution for $\phi_{\rm ir}$ is presented in figure 3 and can be easily seen to interpolate between the UV and IR fixed points. In fact, once the solution $\phi_{\rm ir}$ is found, $W_{\rm ir}$ can be defined as

$$\begin{aligned} W_{\rm ir}(z) &= \int_{-\log 2/\sqrt5}^{z} dw\;\dot\phi_{\rm ir}\left(\phi_{\rm ir}^{-1}(w)\right)-\frac{5}{2^{1/5}\sqrt3}\\ &=\int_{-\infty}^{\phi_{\rm ir}^{-1}(z)} dy\;\dot\phi_{\rm ir}(y)^2-\frac{5}{2^{1/5}\sqrt3}\,. \end{aligned} \tag{2.13}$$

[7]There is only one constant, c, because the other conjugate variable is set to zero by (2.7).
[8]This is defined up to an overall unimportant sign.

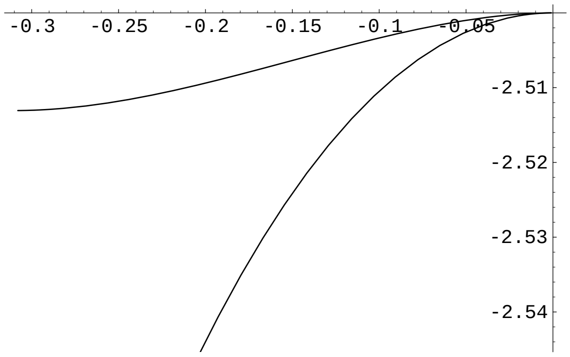

Figure 2: Comparison of the two generating functions $W_{\rm ir}$ and $W_{\rm susy}$ between the IR and UV fixed points. Note that $W_{\rm ir}$ has a second extremum at the IR fixed point, whereas $W_{\rm susy}$ does not.

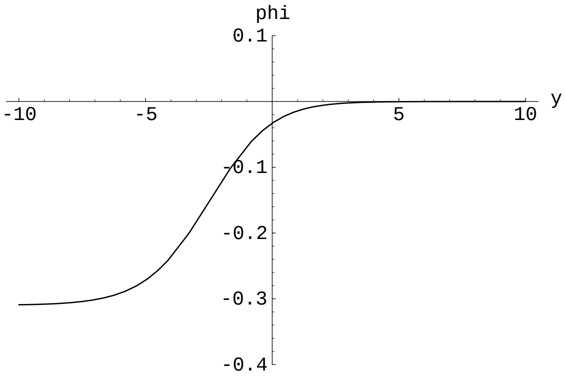

Figure 3: The solution $\phi_{\rm ir}$ connecting the two fixed points plotted against the scale factor y.

The constant in (2.13) is chosen to agree with (2.11) at the IR point. It is interesting to analyze the behaviors of $W_{\rm susy}$ and $W_{\rm ir}$ near the origin. Obviously, $W_{\rm susy}$ is analytic and

$$\begin{aligned} W_{\rm susy}(0) &= -\frac{5}{2}\,, \qquad W'_{\rm susy}(0) = 0\,, \\ W''_{\rm susy}(0) &= -2\,, \qquad W'''_{\rm susy}(0) = \frac{6}{\sqrt{5}}\,, \end{aligned} \tag{2.14}$$

whereas $W_{\rm ir}$ is not analytic, since

$$\begin{aligned} W_{\rm ir}(0) &= -\frac{5}{2}\,, \qquad W'_{\rm ir}(0) = 0\,, \\ W''_{\rm ir}(0) &= -1\,, \quad W'''_{\rm ir}(0) = \infty\,. \end{aligned} \tag{2.15}$$

The two solutions $W_{\rm susy}$ and $W_{\rm ir}$ act as boundaries for a continuum set of solutions that lay between them, all of which have the same behavior as (2.15).[9]

Since the second derivative of W determines whether the behavior of ϕ at $y \to +\infty$ is square-integrable or not, we see that $W_{\rm ir}$ gives rise to a non-square-integrable behavior, thus corresponding to deforming the fixed point lagrangian by $\mathcal{O}_\phi$.

In fact *none* of the other solutions (including the superpotential) is physically acceptable in the region $\phi < 0$ because they would correspond to giving a negative vev to $\mathcal{O}_\phi$, a manifestly positive operator. If we write the asymptotics of ϕ as[10]

$$\phi \approx A\, e^{-2y/r} + B\, e^{-4y/r} \tag{2.16}$$

[9]There are also solutions with only an extremum at the IR point which we do not consider [10].
[10]For simplicity we do not write the polynomial corrections. Also recall that $r^2 = -15/V(0) = 4$

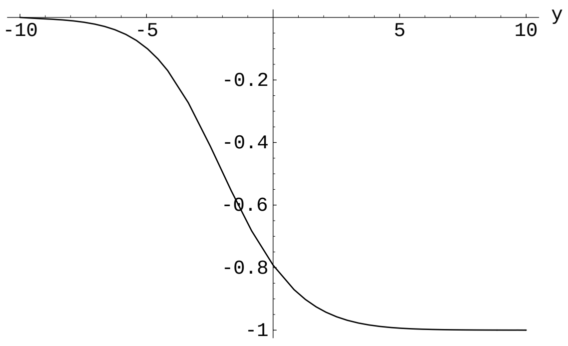

Figure 4: The asymptotic behavior of $\dot{\phi}/\phi$ shows that $\phi \approx e^{-y}$ when $W = W_{\rm ir}$ corresponding to a true deformation by $\mathcal{O}_\phi$.

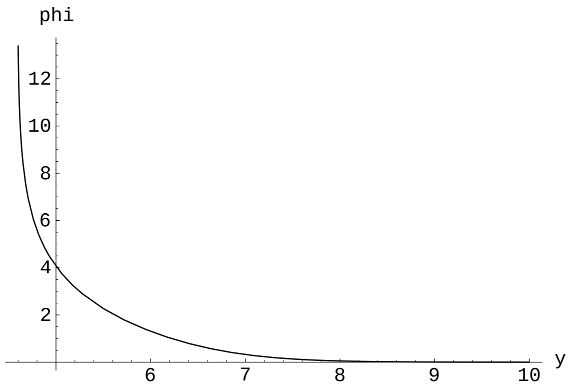

Figure 5: The supersymmetric runaway solution $\phi_{\rm susy}$ corresponding to a non-conformal vacuum where $\langle \mathcal{O}_\phi \rangle > 0$, plotted against the scale factor y in the vicinity of the UV fixed point.

the above analysis shows that $A = 0$ for $W_{\rm susy}$ and non zero for the others. This is shown in figure 4 for the particularly interesting case where the generating function is $W_{\rm ir}$. If we take the case of $W_{\rm susy}$, so that the vev becomes the leading term, we get $B < 0$, since we are studying the region $\phi < 0$. The term B should still remain negative by continuity as we use generating functions laying between $W_{\rm susy}$ and $W_{\rm ir}$ and it will reach zero at $W_{\rm ir}$, precisely as A reaches zero at the opposite end ($W_{\rm susy}$). B corresponds to a vev for $\mathcal{O}_\phi$ and therefore a negative value must be excluded. It would be nice to check numerically that our picture is correct but it is rather difficult to isolate the sub-leading term when $A \neq 0$.

Since the leading exponential behavior for all generating functions is known explicitly, we can be more precise and analyze the differences between positive and negative ϕ. From (2.9) and (2.10), following [13] and doing the asymptotics for large $|\phi|$, one can easily see that the metric has the following behavior, (shifting the singularity to $y = 0$)

$$\phi > 0: \; ds^2 = y^2 \, \eta_{\mu\nu} \, dx^\mu \, dx^\nu + dy^2 \, , \tag{2.17}$$

$$\phi < 0: \; ds^2 = y^{1/8} \, \eta_{\mu\nu} \, dx^\mu \, dx^\nu + dy^2 \, . \tag{2.18}$$

Solution (2.18) corresponds to a naked time-like singularity and our analysis says that it should be excluded. On the other hand, the runaway solution for $\phi > 0$ is acceptable and plotted in the neighborhood of the UV point in figure 5. This solution corresponds to going to new non-supersymmetric vacua where $\langle \mathcal{O}_\phi \rangle > 0$, which for the special case of the superpotential corresponds to a supersymmetric vacuum. The reason we have many acceptable generating functions for $\phi > 0$ is that they simply correspond to different vev's (or vacua), in contrast to the $\phi < 0$ case.

3. Squashing deformations of $d = 4$ supergravity

The final issue we discuss is the behavior of $\mathcal{N} = 1$, $d = 3$ superconformal field theories obtained from solutions of $d = 11$ supergravity [16] on squashed seven-manifolds. These examples are of interest because they involve fields from higher levels of the Kaluza-Klein tower. They were recently considered in [15] — we would like to make a few remarks complementing that analysis.

The two main examples of manifolds allowing for a squashed solution are the seven-sphere and the manifold $N(1,1)$ [17]. Squashed metrics are Einstein metrics, obtained by stretching the original one in some directions. $N(1,1)$ is a particular instance of a class of seven-dimensional Einstein manifolds named $N(p,q)$ [17], which has the peculiarity of preserving $\mathcal{N} = 3$ supersymmetries, whereas its squashed version has $\mathcal{N} = 0$.[11] By the AdS/CFT correspondence all these solutions ought to correspond to some conformal limits of three-dimensional QFT representing the degrees of freedom living on M2-branes placed at the apex of the cone over the compactification manifolds [18].

By the standard procedure, one reads off the global symmetries ("flavor" and R-symmetry) from the isometries of the corresponding solution. Once an appropriate guess for the gauge group is made, it is possible to get a detailed mapping between the operators and the fields of the KK spectrum, based on matching supersymmetry representations. The theory dual to S^7 is a 3-dimensional $\mathcal{N} = 8$ CFT with SO(8) R-symmetry group, while $N(1,1)$ gives rise to a CFT with SU(3) $\times$ SU(2) global symmetries and supercharges transforming in the **3** of SU(2). Moreover, the situation is more difficult for squashed solutions. Having $\mathcal{N} = 1$ in three dimensions there is no R-symmetry and the usual procedure does not apply straightforwardly. The case of $N(1,1)$ is particularly puzzling as squashed and unsquashed solutions share the same global symmetries.

To study the possibility of having domain-wall solutions interpolating between such theories one considers a truncation of the Kaluza-Klein spectrum and derives an effective four-dimensional action for the non-zero fields. The potential for the sphere is known from the work of [19] in terms of two scalars u and v appearing in the eleven-dimensional metric as

$$ds^2 = e^{-7u}\, ds^2\,(AdS_4) + e^{2u+3v}\, ds^2\,(\text{base}) + e^{2u-4v}\, ds^2\,(\text{fibre})\,, \tag{3.1}$$

where the seven-sphere is thought of as a S^3 fibration over the base S^4. The potential for the squashed $N(1,1)$ has been given in terms of four scalars in [20] but for our purposes it is sufficient to repeat the computation of [19] using (3.1) where now the base manifold is $\mathbb{CP}^2$ and the fiber is $\mathbb{RP}^3$, thus obtaining a potential also dependent

[11] However, the orientation-reversed or "skew-whiffed" solution is supersymmetric, with $\mathcal{N} = 1$ [14].

only on two scalars. In both cases the potential can be written as

$$V(u,v) = \lambda\, e^{-9u} \left(\alpha\, e^{4v} - e^{3v} - \frac{1}{32\alpha}\, e^{-10v} \right) + 2\, Q^2\, e^{-21u}\,, \tag{3.2}$$

where Q is the Page charge and, for the sphere, $\alpha = -1/8$ and $\lambda = 48$, whereas, for $N(1,1)$, we have $\alpha = -1/16$ and $\lambda = 24$. Amusingly, all the physical quantities, such as the conformal dimensions for the operators, turn out to be independent of α and λ.

The potential (3.2) has two fixed points but the field u always describes a non-renormalizable (irrelevant) operator. From the equivalent mechanical problem, the flow between these two points would have to connect a maximum of $-V$ to a saddle point of $-V$, clearly an unstable situation — contrary to the situation occurring for some flows in $d = 5$ gauged supergravity, where the "particle" rolls along a valley from a saddle point to a minimum.

As mentioned in the introduction, if the RG equations where truly first order, one could argue from general theorems that there must still be a critical line connecting the points. However, the equations expressed in terms of the potential are second order and the existence of this solution is not guaranteed in this case. After some numerical tests we now believe that there is no such flow.

The potential (3.2) has another peculiar property: it is possible to find explicitly one generating function W that has one critical point at the squashed solution. The function is[12]

$$W(u,v) = -\frac{1}{\sqrt{8}}\, e^{-\frac{9}{2}u} \Big(3\, e^{2v} + 6\, e^{-5v} - |Q|\, e^{-6u} \Big)\,, \tag{3.3}$$

which is a solution to

$$V = \frac{16}{63}\, (\partial_u\, W)^2 + \frac{8}{21}\, (\partial_v\, W)^2 - 12\, W^2\,. \tag{3.4}$$

At first, it seems rather counterintuitive that the point with *less* supersymmetries should appear as an extremum, but we must remember that we are not dealing with a gauged supergravity, where only low-lying KK excitations are included. Still, it is tempting to believe that the solution associated with W describes different *supersymmetric* vacua of the theory. As a check one can show, expanding W near its critical point, that the solution corresponds to the operator associated to v getting a vev — more specifically $v \sim \exp(-5\, y/3\, r)$. There is a choice between a theory in which the conformal dimension of the operator is $\Delta = 5/3$ or $\Delta = 4/3$, which is also allowed [4]. Finally, one finds that the runaway solution also satisfies the criterion [13] of boundness from above of the potential. Hopefully, further investigations will reveal if even these models consistently describe holographic RG flows.

[12]We take the case of the sphere for concreteness. For the $N(1,1)$ case the factor 3 in front of e^{2v} is changed to $3/2$.

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